Solutions to Recitation 11/24

1&2. X = # of heads in 2 tosses

outcomes	HH	HT	TH	TT
x	2	1	1	0
$p(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

expected value of $X = E(X)$

$$= \sum_{\text{all } x} x\,p(x)$$

$$= 2\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{4}\right) + 0\left(\tfrac{1}{4}\right)$$

$$= 1$$

Note: this is not a probability, it means the expected value of X is 1 (we expect to get 1 heads).

3. X_1 = # of heads on 1 coin toss

outcomes	H	T
x_1	1	0
$p(x_1)$	$\frac{1}{2}$	$\frac{1}{2}$

$$E(X_1) = \sum_{\text{all } x} x_1 p(x_1) = 1\left(\tfrac{1}{2}\right) + 0\left(\tfrac{1}{2}\right) = \tfrac{1}{2}$$

We expect ½ heads. The values don't always make perfect logical sense.

4. X_1 = # of heads on first coin toss in a total of 2 tosses

outcomes	HH	HT	TH	TT
x_1	1	1	0	0
$p(x_1)$	¼	¼	¼	¼

$$E(X_1) = \sum_{\text{all } x} x_1 p(x_1) = 1(\tfrac{1}{4}) + 1(\tfrac{1}{4}) + 0(\tfrac{1}{4}) + 0(\tfrac{1}{4})$$
$$= \tfrac{1}{2}$$

5. Additivity Property of Expectations

$$E(aX + bY + c) = aE(X) + bE(Y) + c$$

a, b, c are constants , X, Y are random variables

6. $$\text{net} = 1.3X + Y - (1.3)(.28)X - .17Y + (1.3)(1.4) + 3.2$$
$$= [1.3 - (1.3)(.28)]X + [1 - 0.17]Y + [(1.3)(1.4) + 3.2]$$
$$= 0.936X + 0.83Y + 5.02$$

Find E(net). (apply additive property)

$$E(\text{net}) = E(0.936X + 0.83Y + 5.02)$$
$$= 0.936E(X) + 0.83E(Y) + 5.02$$
$$= 0.936(23.69) + 0.83(27.94) + 5.02$$
$$= 50.38404 \text{ million}$$

7. unbalanced coin :

$P(H) = 0.54$, $P(T) = 1 - 0.54 = 0.46$

X = # of heads in 2 independent tosses

outcomes	HH	HT	TH	TT
x	2	1	1	0
$p(x)$	$(.54)(.54)$	$(.54)(.46)$	$(.46)(.54)$	$(.46)(.46)$

$$E(X) = \sum_{\text{all } x} x\,p(x) = 2(.54)^2 + 1(.54)(.46) + 1(.46)(.54) + 0(.46)^2$$
$$= 1.08$$

8. Same situation as Q7.

X = # of heads in 2 independent tosses

x	2	1	0
$p(x)$	$.54^2$	$(.54)(.46) + (.46)(.54)$	$.46^2$

there are 2 ways to get 1 Head
HT and TH

$$E(X) = \sum_{\text{all } x} x\,p(x) = 2(.54)^2 + 1[(.54)(.46) + (.46)(.54)] + 0(.46)^2$$
$$= 1.08$$

Q8 gets the same result as Q7, the difference is in the setup of the problem.

9. X_1 = # of heads in 1 toss

x_1	1	0
$p(x_1)$	.54	.46

$E(X_1) = 1(.54) + 0(.46) = .54$

10. Setup the problem as follows (to use additivity property):

X_1 = # of heads in 1^{st} toss
X_2 = " " " " 2^{nd} toss
$\vdots$
X_{100} = " " " " 100^{th} toss

$E(X_1) = E(X_2) = \cdots = E(X_{100}) = .54$ (result from Q9)

Want to find: $E(\text{net})$

$$\text{net} = X_1 + X_2 + \cdots + X_{100}$$

$$\begin{aligned} E(\text{net}) &= E(X_1 + X_2 + \cdots + X_{100}) \\ &= E(X_1) + E(X_2) + \cdots + E(X_{100}) \\ &= .54 + .54 + \cdots + .54 \\ &= .54(100) \\ &= 54 \end{aligned}$$

In 100 tosses, we expect 54 heads.

More Important Properties:

If X & Y are "Independent",

$$E(XY) = E(X)E(Y)$$

Variance of X:

$$Var(X) = E(X^2) - \left(E(X)\right)^2$$

SD of X:

$$SD(X) = \sqrt{Var(X)}$$

11. unbalanced coin:

$P(H) = 0.54$, $P(T) = 0.46$

X_1 = # of heads in 1 toss

x_1	1	0
$p(x_1)$	.54	.46

$$E(X_1) = \sum_{\text{all } x} x_1 p(x_1) = 1(.54) + 0(.46)$$
$$= .54$$

12. Same setup as Q11, but want to find $E(X_1^2)$.

$$E(X_1^2) = \sum_{\text{all } x} x_1^2 p(x_1) = 1^2(.54) + 0^2(.46)$$
$$= .54$$

13. Find $Var(X_1)$. (use results from Q11 & Q12)

$$Var(X_1) = E(X_1^2) - (E(X_1))^2$$
$$= .54 - .54^2$$
$$= 0.2484$$

14. Same as Q10.

$E(net) = 54$

15. The coin is tossed 100 times. Each toss ~~is~~ is independent.

$$net = X_1 + X_2 + \cdots + X_{100}$$

Find $Var(net)$.

<u>Property of Variance</u>: if X & Y are independent

$$Var(X+Y) = Var(X) + Var(Y)$$

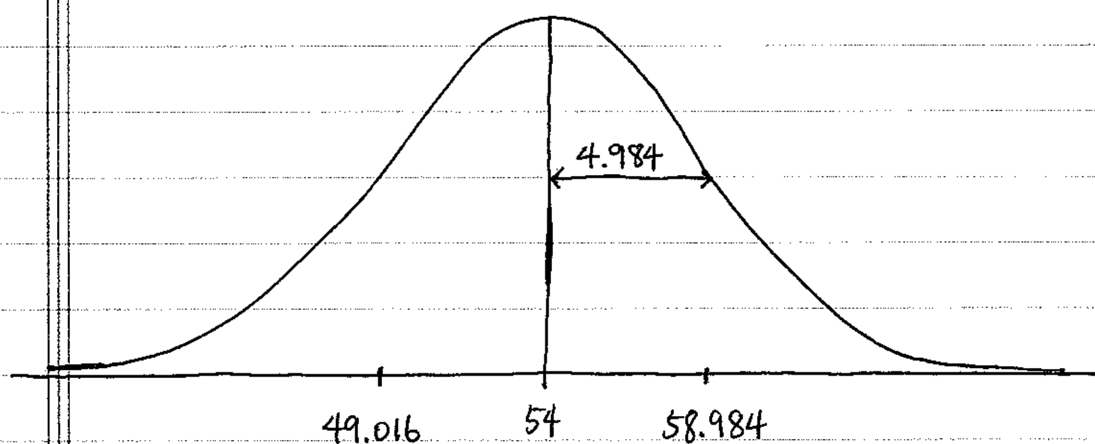
$$Var(X_1) = Var(X_2) = \cdots = Var(X_{100}) = 0.2484 \text{ (from Q13)}$$

$$Var(net) = Var(X_1 + X_2 + \cdots + X_{100})$$
$$= Var(X_1) + Var(X_2) + \cdots + Var(X_{100}) \quad \text{(independence)}$$
$$= 0.2484 + 0.2484 + \cdots + 0.2484$$
$$= 0.2484(100)$$
$$= 24.84$$

16. $E(X) = 54$

$Var(X) = 24.84$

$SD(X) = 4.984$



68% interval : $(49.016, 58.984)$